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Light quark masses from the lattice*

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A completely non-perturbative estimate is given for the u/d and strange quark masses in quenched QCD using $O(a)$ improved fermions and, for comparison, Wilson fermions. For improved fermions we find $m_{u/d}^{\overline{MS}}(\mu = 2\,\mathrm{GeV}) = 4.4(2)\,\mathrm{MeV}$, $m_s^{\overline{MS}}(\mu = 2\,\mathrm{GeV}) = 105(4)\,\mathrm{MeV}$ when using r_0 to set the physical scale.

1. THE LATTICE APPROACH

Lattice methods allow, in principle, an 'ab initio' calculation of the fundamental parameters of QCD, among them the quark masses. These, not being asymptotic states of the Lagrangian, need to be defined by giving the scheme $\mathcal{S}$ and scale M. In this brief report we give our recent results for the light quark masses. Further details may be found in [1].

The starting point is the PCAC quark mass $a\widetilde{m}_q$, which can be determined from the ratio of the correlation functions

$$a\widetilde{m}_{q_1} + a\widetilde{m}_{q_2} \overset{t \gg 0}{=} \frac{\langle \partial_4 \mathcal{A}_4^{q_1 q_2}(t)\mathcal{P}^{q_1 q_2}(0)\rangle}{\langle \mathcal{P}^{q_1 q_2}(t)\mathcal{P}^{q_1 q_2}(0)\rangle},$$

where $\mathcal{A}$ and $\mathcal{P}$ are the axial and pseudoscalar currents, respectively, for possibly non-degenerate quarks q_1 and q_2. Expanding $a\widetilde{m}_{q_1} + a\widetilde{m}_{q_2}$ (with expansion coefficients $\widetilde{Y}$, $\widetilde{c}$) and the pseudoscalar mass $am_{PS}^{q_1 q_2}$ (with expansion coefficients Y_{PS}, c_{PS}) in terms of the masses am_{q_i} ($\equiv \frac{1}{2}(1/\kappa_{q_i} - 1/\kappa_c)$, $i = u/d, s$) and renormalising to the 'renormalisation group invariant' (RGI) form gives

$$r_0 m_{u/d}^{RGI} = c_a^*(r_0 m_{\pi^+})^2 + c_b^*(r_0 m_{\pi^+})^4,$$

and

$$r_0 m_s^{RGI} = c_a^*\left[(r_0 m_{K^+})^2 + (r_0 m_{K^0})^2 - (r_0 m_{\pi^+})^2\right] + c_b^*\left[(r_0 m_{K^+})^4 + (r_0 m_{K^0})^4 - (r_0 m_{\pi^+})^4\right].$$

We have defined $m_{u/d}^{RGI} = (m_u^{RGI} + m_d^{RGI})/2$, and have ignored any small corrections due to electromagnetic effects. The 'force' scale [2] has been used to set the physical scale and the $c^* \equiv \lim_{g_0 \to 0} c$ coefficients are given by

$$c_a = F\left[\frac{\widetilde{Y}}{Y_{PS}}\right]\left(\frac{r_0}{a}\right)^{-1},$$

$$c_b = F\left[\frac{\widetilde{Y}}{Y_{PS}}\right]\left[\frac{-c_{PS}}{Y_{PS}}\left(\frac{r_0}{a}\right)^{-2}\right]\left(\frac{r_0}{a}\right)^{-1},$$

where $F \equiv \Delta Z^{\mathcal{S}}(M)\widetilde{Z}_m^{\mathcal{S}}(M)$ with $\widetilde{Z}_m^{\mathcal{S}}(M)$ the renormalisation constant and $\Delta Z^{\mathcal{S}}(M)$ converting the renormalised mass to the RGI mass m^{RGI}. Our convention is to use

$$[\Delta Z_m^{\mathcal{S}}(M)]^{-1} = \left[2b_0 g^{\mathcal{S}}(M)^2\right]^{\frac{d_{m0}}{2b_0}} e^{\int_0^{g^{\mathcal{S}}(M)} d\xi \left[\frac{\gamma_m^{\mathcal{S}}(\xi)}{\beta^{\mathcal{S}}(\xi)} + \frac{d_{m0}}{b_0 \xi}\right]}.$$

*Talk given by R. Horsley at Lat99, Pisa, Italy.

(In the $\overline{MS}$ scheme $\beta^{\overline{MS}}$ and $\gamma_m^{\overline{MS}}$ are known to four loops.)

To determine the expansion coefficients we shall consider degenerate quarks,

$$\begin{aligned} a\widetilde{m}_q &= \widetilde{Y}\,[1 + (\widetilde{c} + d)am_q]\,am_q, \\ (am_{PS})^2 &= Y_{PS}\,[1 + (c_{PS} + d)am_q]\,am_q, \end{aligned}$$

giving

$$\frac{a\widetilde{m}_q}{(am_{PS})^2} = \frac{\widetilde{Y}}{Y_{PS}}\left[1 + \left(\frac{\widetilde{c} - c_{PS}}{Y_{PS}}\right)(am_{PS})^2\right].$$

While the constant term on the r.h.s. of this equation is sufficient to find c_a, the gradient term does not fully determine c_b. (We shall assume in the following that $\widetilde{c}$ is small [1].) In Fig. 1 we show this ratio for $O(a)$ improved fermions. Below $m_q \sim m_s$ there are significant

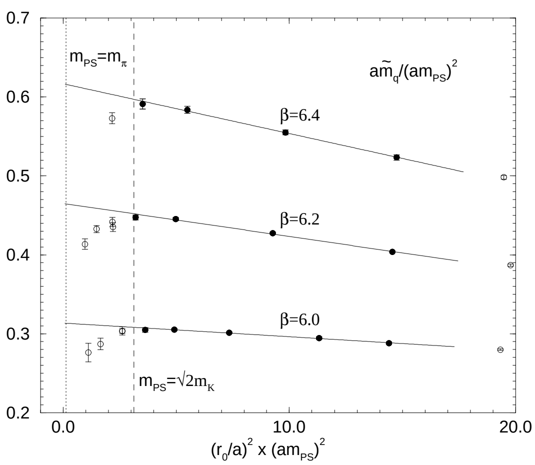

Figure 1. *The ratio $a\widetilde{m}_q/(am_{PS})^2$ against $(r_0/a)^2 \times (am_{PS})^2$ for $\beta = 6.0,\ 6.2,\ 6.4$. The filled circles are the data used in the fits. The dashed line (~ 3.13) corresponds to $m_{PS} = \sqrt{2}m_K$, while the dotted line (~ 0.125) is m_π.*

deviations from linearity, due to the presence of chiral logarithms. For large quark masses, we expect non-linear terms to appear. From the figure a safe linear region would seem to be $m_s \lesssim m_q \lesssim \frac{1}{3}m_c \sim 3m_s$ (we have $2(r_0 m_D)^2 \sim 44.9$).

Similar results also hold for Wilson fermions (although there we only have two β ($\equiv 6/g_0^2$) values, namely 6.0 and 6.2).

We also need $F(g_0)$. For $O(a)$ improved fermions this was recently achieved by the ALPHA collaboration [3], using the Schrödinger Functional formalism, giving $F(g_0)$ for $6.0 \leq \beta \leq 6.5$. For Wilson fermions [4] we used the method proposed in [5], which mimics perturbation theory in a MOM scheme by considering amputated quark Green's function, with an operator insertion. By 'converting' the MOM scheme to the $\overline{MS}$ scheme [6], we have found $F(g_0)$ at $\beta = 6.0,\ 6.2$.

2. CONTINUUM RESULTS

We now have all the pieces necessary to compute c_a (and c_b approximately) and to extrapolate to the continuum limit. In Fig. 2 we plot our results for c_a for $O(a)$ improved

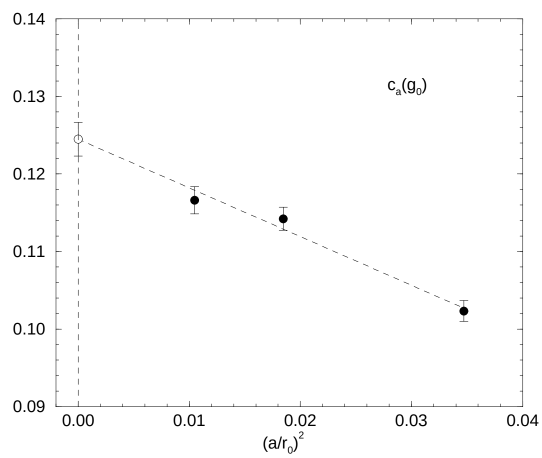

Figure 2. *The continuum extrapolation for c_a for $O(a)$ improved fermions.*

fermions. Similar results also hold for c_b and for Wilson fermions. (The latter are, of course, extrapolated in a rather than a^2.) Using these results and the physical kaon and pion masses, we find

$$m_{u/d}^{RGI} = 6.1(2)\,\mathrm{MeV}, \quad m_s^{RGI} = 146(4)\,\mathrm{MeV}.$$

(Note that we have used $r_0 = 0.5\,\mathrm{fm}$ to set the scale. Other choices can lead to an $O(10\%)$ difference.) For Wilson fermions we have $m_{u/d}^{RGI} = 5.3(8)\,\mathrm{MeV}$, $m_s^{RGI} = 121(20)\,\mathrm{MeV}$. These results are somewhat lower than the

$O(a)$ improved numbers but with far larger error bars. Note that as we only have two values of β, this makes a continuum extrapolation more difficult. Also the number of κ values used and the size of the data sets are smaller than for $O(a)$ improved fermions. Nevertheless, within a one-standard deviation the results are in agreement.

In the $\overline{MS}$ scheme at the 'standard' value of $\mu = 2\,\mathrm{GeV}$ we find for $O(a)$ improved fermions

$$m_{u/d}^{\overline{MS}} = 4.4(2)\,\mathrm{MeV}, \quad m_s^{\overline{MS}} = 105(4)\,\mathrm{MeV}.$$

The equivalent Wilson results are 3.8(6) MeV and 87(15) MeV for the u/d and strange quark masses respectively.

For the $m_{u/d}$ quark mass result we have simply extrapolated the fits for the strange quark mass result. The mass ratio $m_s/m_{u/d}$ for $O(a)$ improved fermions is ≈ 23.9, which is very close to the value given in leading order chiral perturbation theory, namely $(m_{K^+}^2 + m_{K^0}^2 - m_{\pi^+}^2)/m_{\pi^+}^2 \approx 24.2$. This is simply because $|c_b^*| \ll |c_a^*|/(r_0 m_K)^2$, and so the second term in the Taylor series expansion is negligible. The mass ratio is then independent of c_a^*.

3. COMPARISON WITH OTHER RECENT RESULTS

We shall now briefly compare our results (QCDSF) with other recent quark mass determinations (published since the last lattice conference). In Fig. 3 we show the results from [7–9] and [10]. Closest to the method used here is [10] (ALPHA/UKQCD). We have not converted the different physical scales used in [7–9] to the r_0 scale. The quark masses are determined from the pion and kaon. Ref. [7] uses $O(a)$ improved fermions mainly at $\beta = 6.2$ and the method of [5] for the renormalisation constants, while [8] (JLQCD) uses staggered Wilson fermions at three β values and also [5] for the renormalisation constants. Ref. [9] (CP-PACS) uses Wilson fermions (with four β values and a one-loop perturbative renormalistion factor). Within a rough 10% band there is agreement.

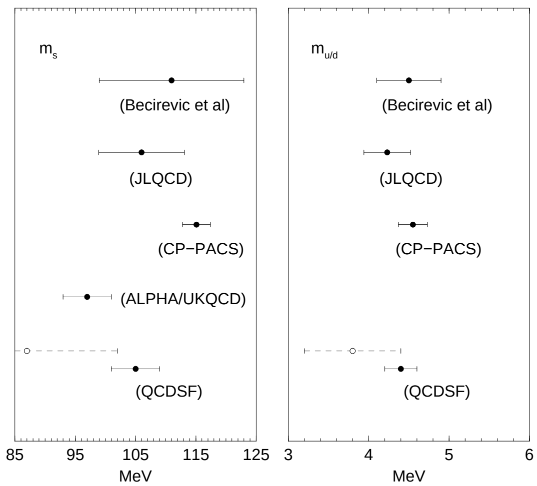

Figure 3. *Some recent quark mass results in the $\overline{MS}$ scheme at a scale of $\mu = 2\,GeV$. The references are given in the text. Our Wilson quark mass determination is denoted by dashed error bars.*

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